Pulse distributions and tracking in segmented detectors

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Abstract

A study of the performance of a cylindrical HPGe detector segmented in 25 segments is presented. It is based on simulations made with the computer code GEANT and focuses on the reconstruction of a $\gamma$-ray path. The effects of the segmentation are initially discussed in terms of Doppler correction. The role of the pulse shape analysis and its effects on tracking algorithms are discussed as a function of the capability to reconstruct a $\gamma$-ray path when multiple signals (direct and induced) are present in a single segment. It is found that it is critical to identify correctly at least two signals in a segment in order to have a reasonable efficiency and Compton suppression in the spectra and to make a good use of this type of detectors. © 2002 Elsevier Science B.V. All rights reserved.


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1. Introduction

A great deal of progress in the study of nuclear structure has been made through $\gamma$-spectroscopy measurements employing large detection facilities based on germanium detectors. With the largest existing germanium arrays, EUROBALL [1] and GAMMASPHERE [2], not only the physics of high spins has been extensively addressed, but also the isospin degree of freedom has been partly investigated through the $\gamma$ decay of nuclei far from the stability line. This line of research is now pushed to more extreme cases making use of the available radioactive beams, which are presently opening new frontiers in nuclear structure. Gamma spectroscopy measurements with radioactive beams present different technical problems as compared with those of stable beams and the discussion on the best possible solutions is currently underway. In order to make feasible the study of $\gamma$-transitions emitted by exotic nuclei at a rather high velocity and with a rather low beam intensity, it is essential that the new arrays have higher efficiencies and degree of segmentation as compared to the existing set-ups. Moreover, the high velocities imply large values of the Doppler shift and Doppler broadening. It is, therefore, very important to have a good definition of the $\gamma$ emission angle with respect to the direction of the emitting source in these measurements. The arrays MINIBALL [3] and EXOGAM [4] presently under construction consist of germanium detectors
with the front face segmented in a number of sectors in order to take care of the problem of the Doppler broadening. These central questions are presently discussed among the nuclear structure community also in view of the construction of new arrays.

In connection with the increase of efficiency, one important and clear point is that the design on which the large arrays EUROBALL and GAMMASPHERE were based cannot be simply extended. In fact, in these arrays the BGO Compton suppression shields, with which one obtains a full energy peak (FEP) to total ratio ($P/T$ ratio) of about 60%, cover approximately 50% of the solid angle. If one simply increases the amount of germanium and reduces or removes the BGO scintillators, then one would obtain a very small $P/T$ ratio and FEP efficiency due to the scattering of the $\gamma$-rays between different detectors. A possible solution is that of using only germanium detectors, increasing the part covered by the germanium material up to nearly 100%, with a high electrical segmentation and some tracking procedure. Now, the signals coming from each interaction points have to be identified. Then a pulse shape analysis has to deduce the position and energy deposited corresponding to each interaction points. Finally, a tracking algorithm has to reconstruct the path of the $\gamma$-ray and decide whether it corresponds to a FEP event or not, to improve the peak/total ratio and efficiency. However, this is not as simple as it looks, particularly because the segmentation cannot be sufficiently small to have a single interaction of the incident $\gamma$-ray in a given segment. Moreover, in most experiments, several $\gamma$-rays are emitted that could hit simultaneously the same section of the detector.

Several initiatives focusing on segmentation have already started both in Europe (TMR project [5,6], MARS [7]), and in USA (GRETA [8], MSU [9]). Only a few prototypes of compact and cylindrical 24–36 segmented HPGe detectors are already available [7–9]; consequently, most of the work has been done on simulations [10–17]. Besides, a properly working segmented detector is only the first and probably the easiest step to the complete tracking of an incident $\gamma$-ray. Some algorithms have been proposed and are under development to recognise several interactions in a given segment [18,19], but they have a limited capability of disentangling overlapping interactions and it is therefore important to estimate the relative consequences on the final performances of a tracking procedure in a HPGe detector.

In this paper, we present and discuss the simulations of $\gamma$-ray interacting in a segmented detector of cylindrical shape segmented in 25 segments. It is important to mention that while the tracking algorithms have been discussed in several works [13–16], here, in contrast, the tracking efficiency is investigated in terms of the capability to reconstruct events with multiple interactions in a given segment. In the first section, we consider the worst case where no Pulse Shape Analysis (PSA) is available, namely neither the number of interaction points nor their position inside the detector segment is available. The general performance of such a segmented detector, with no PSA, has been simulated and FEP efficiency and $P/T$ ratio have been extracted for one emitted $\gamma$-ray. In the second part of the paper, the results of the tracking algorithm when the PSA fails to correctly disentangle two, three or more signals are discussed. A minimum performance of the PSA is then extracted.

2. Geometrical details, simulation inputs and tracking algorithms

The simulations of $\gamma$-ray interactions were made for a detector with a cylindrical geometry. The cylinder has a diameter of 7.2 cm and a length of 9 cm, with an inner hole of 6.5 cm length and 1 cm diameter. It corresponds to the MARS prototype [7] and is segmented in 25 segments: four identical slices in the depth, each segmented in 6 sectors and an additional circular segment with a diameter of 1 cm centred on the front face of the crystal. The source of $\gamma$-rays has been placed at a distance of 15 cm on the axis of the cylinder.

The computer code GEANT [21] has been employed. A position resolution of 1 mm and a resolving distance of 5 mm, namely all the energy deposited within 5 mm being summed up, have been applied.
All the simulations are made considering that only one $\gamma$-ray has been emitted. In most $\gamma$-spectroscopy applications, one deals with energies in the interval 0.2–3 MeV. In this range, the $\gamma$-ray mostly interacts with the detector through Compton mechanism. Incident $\gamma$-rays lose energy by scattering several times on the electrons of the crystal until their energy has become low enough for photoelectric absorption.

Two different tracking algorithms have been considered. One is called “backtracking” [14]. This algorithm selects interaction points associated with an energy deposition close to 100 keV as they are likely to correspond to a photoelectric absorption of the $\gamma$-ray and consequently will be the last interaction point of the trajectory. Then, it traces the track back, step by step using the Compton scattering formula and the cross-sections for the photo and Compton effects, down to the source of the $\gamma$-ray. The other one is very simple as it takes advantage of the present case where only one $\gamma$-ray is emitted (one-$\gamma$-tracking) [16]. A test of all permutations of the identified interaction points (position and energy deposited) is carried out against the Compton scattering formula, with a $\chi^2$ procedure, in order to distinguish the acceptable sequences (FEP events) from those that must be rejected because of an incomplete absorption of the $\gamma$-ray. Of course such an algorithm could not be applied to realistic cases with more than one $\gamma$-ray per event; but in the present simple case, it can give an upper estimate of the tracking efficiency.

3. Basic performance of segmented detectors without PSA

In contrast to the standard Ge detectors providing a single electric signal with the amplitude proportional to the deposited energy, for a segmented detector, each event is characterised by a pattern of signals. This pattern consists of signals with a net charge (whose amplitude is proportional to the deposited energy) associated to the segments activated by the $\gamma$-ray interaction and of “zero charge” signals. The latter are induced signals present in the segments near the activated ones as a result of the electrical segmentation. In this paragraph, we study the performance of the detector in the case in which no PSA is made and therefore only the pattern of signals is available. This implies that one measures the total energy deposited in a segment and localises the hit at the geometrical centre of the segment. With these given conditions one can evaluate not only the Doppler correction (affecting the FWHM of the FEP) and the $P/T$ parameter, but one can also attempt a very simple tracking of the event.

For the Doppler correction, which is necessary to have a good energy resolution, the adopted procedure takes into account the fact that for $\gamma$-ray energies in the interval 0.2–3 MeV, Compton scattering is the main interaction mechanism. In Compton scattering, because of momentum conservation, the $\gamma$-ray cannot give all its energy to the crystal electron and a minimum energy ranging from 100 to 256 keV must be given to the scattered photon. Consequently, a photon with $E_\gamma < 200–500$ keV which interacts more than once in the detector is likely to deposit less than 50% of its total energy in its very first interaction and consequently, in the first interaction segment. For photons with $E_\gamma > 500$ keV, instead, the first interaction segment is likely to be the one with the highest deposited energy. Such a general argumentation can be used to determine the incident direction of the $\gamma$-ray by identifying the segment where the $\gamma$-ray first interacted with the germanium crystal (the one with the highest energy for $E_\gamma \geq 500$ keV and the one with lowest deposited energy for $E_\gamma < 500$ keV). Once the first hit segment is recognised, the associated angle can be used to correct the Doppler effect on the measured energy. Such an algorithm has already been used successfully with a MINIBALL CLUSTER detector [3] and an EXOGAM CLOVER detector [4].

Fig. 1 shows the calculated probability that the first hit segment corresponds to the segment which has the highest (if $E_\gamma \geq 500$ keV) or the lowest (if $E_\gamma < 500$ keV) deposited energy for a $\gamma$-ray ranging from 250 keV to 10 MeV. Only FEP events have been considered. The plot shows that such a simple method works pretty well, in fact the first hit segment is correctly recognised in about 80% of the events. One can note the jump around 500 keV.
reflecting the fact that at this energy the identification criterion changes. The corresponding calculated FWHM of the photo-peak without and with the Doppler correction is shown in Fig. 2 as a function of γ-ray energy. The calculation has been done for a detector placed at 90° relative to a source with β = 0.15. In the simulation, the intrinsic energy resolution of the detector has not been taken into account.

As stated in the introduction, only a small fraction of photons of 1 MeV deposit their whole energy in the HPGe detector. Instead, most of them escape from the crystal after few interactions giving rise to a large background. The quality of the measured spectrum can be quantified through the $P/T$ parameter, namely the ratio of events depositing the full energy over all the detected events. An ideal detector with 100% full energy peak efficiency would have $P/T = 1$. Non-segmented HPGe detectors with Compton suppression BGO shields have a $P/T$ of ~60%.

Full energy peak events correspond on an average to events with a higher number of interaction points so that by selecting the events, which trigger more than one segment, it is possible to improve the $P/T$ of the detector up to 80%. This is illustrated in Fig. 3 where the $P/T$ is plotted as a function of the number of hit segments (with a net charge) for different energies of the γ-rays. The improvement of the $P/T$ ratio by considering those events with a large number of hit segments is very good, but this is done at the price of a reduction (from 30% to 70%) of the total FEP efficiency. This limit is intrinsic to all techniques that aim at improving the $P/T$ ratio, including the sophisticated tracking techniques like those discussed in the second part of this paper.

In the present situation, even though without any PSA, one has no information on the position of the interaction points, it is still possible to apply some tracking procedure by assuming that the single interaction point is located in the segment centre. For this purpose we have used
the one-γ-tracking analysis technique mentioned above. This technique requires the evaluation of a \( \chi^2 \)-value that should be 0 in the ideal case of a perfect fit with the Compton expression. However, because of the uncertainty in position and energy, the \( \chi^2 \)-value is larger than 0. The purpose of the present simulation is to establish the upper limit \( \chi^2_{\text{upper}} \) which represents a reasonable compromise to get simultaneously a good \( P/T \) ratio and tracking efficiency for the good events which are defined as \( \chi^2 < \chi^2_{\text{upper}} \). In fact, low values of the \( \chi^2_{\text{upper}} \) maximise the \( P/T \) ratio while higher values maximise the reconstruction efficiency, [17].

Fig. 4 shows the simulated energy spectrum together with the spectra obtained from the one-γ-tracking analysis. Two cases are shown with different values of \( \chi^2_{\text{upper}} \). The corresponding reconstruction efficiencies and \( P/T \) ratios are shown in the left part of Fig. 5. The lower panel displays the percentage of FEP events which were recognised as such. The upper panel shows the simulated \( P/T \) ratio. The plots clearly show that, without the position of the interaction points, more stringent conditions (smaller value of \( \chi^2_{\text{upper}} \)) do not improve the \( P/T \) ratio, but still reduces the efficiency. The fact that the \( P/T \) ratio does not improve shows that the events selected by the tracking algorithm are only a random sample of the input data. In other words, all the useful information to recognise FEP events has been lost with the lack of position determination.

The results are essentially identical with the use of the backtracking algorithm, see right panels in Fig. 5. This algorithm, described in detail in Ref. [14], requires the evaluation of a figure of merit \( w \) that reflects the likelihood of identifying a FEP event. More specifically, high values of \( w \) maximise the \( P/T \) ratio, while lower values maximise the reconstruction efficiency so that good events will be defined by \( w > w_{\text{lower}} \).

### 4. Tracking and hits

In this section, we study the performance of the detector in the case where PSA is able to localise the position of the hits within a segment. This implies that one has not only the information on the amount of energy deposited but also the number of interactions and their relative position within each segment. This is the basic input of any tracking algorithm, and the final objective is the reconstruction of the incident γ-ray trajectory in
order to check whether it corresponds to an FEP \( \gamma \)-ray or not and eventually to determine with a high precision its incident direction.

In fact, it has been shown \cite{11} that, depending on the degree of segmentation of the detector, the probability that a \( \gamma \)-ray interacts more than once in a single segment cannot be neglected. Clearly, a high degree of segmentation would reduce the number of individual interactions and signals per segment as well as the uncertainty in the position measurement. The smaller the segments are, the less sophisticated the PSA needs to be. Unfortunately, more than hundreds of segments are needed in a single crystal to sensitively reduce the probability to have more than one signal in each segment of a \( \gamma \)-trajectory \cite{11}.

To quantify the imperfect pulse analysis, we define \( N_s \) as the number of signals (direct or induced) which overlaps in a segment and \( N_{\text{max}} \), the maximum number of signals that one can disentangle. The quantity \( N_{\text{max}} \) depends on the capability of the PSA. It is equal to 2 for the present developments that aim at identifying two signals per segment \cite{20}. However, in this work, we do not limit our investigation only to the effects of a partially wrong identification of the interaction points on the tracking efficiency to the case \( N_{\text{max}} = 2 \), but we also consider larger values. This will tell us how much could be gained with further PSA developments. Table 1 gives the percentage of \( \gamma \)-events for which \( N_s \leq N_{\text{max}} \), namely, the percentage of the \( \gamma \)-events for which we expect to have a correct determination of the position and deposited energy of each interaction points. The values have been taken from Ref. \cite{11} relative to a 1 MeV \( \gamma \)-ray impinging on a 6 \( \times \) 6-segmented detector.

Three different cases have been considered: for each interaction point only the direct signal (column A), the direct and the two strongest induced signals (column B) or the direct and all the induced signals (column C) have to be taken into account by PSA.

From Table 1, one can see that out of the total number of \( \gamma \)-events, 48\% has only one interaction in the hit segments (\( N_s = 1 \)) and consequently, can be reconstructed even with the simplest PSA identifying one hit per segment (\( N_{\text{max}} = 1 \)). In reality, one cannot neglect induced signals since they are essential to determine the position of the single interactions. Therefore, only 14.6\% (strongest induced signals only) or 8.4\% (all induced signals) out of the total number of \( \gamma \)-events will be reconstructed correctly for \( N_{\text{max}} = 1 \). In other words, in about 90\% of the \( \gamma \)-events, there is one segment with a superposition of pulses. Since the present developments of the PSA correspond to \( N_{\text{max}} = 2 \), it is important to discuss in some detail the corresponding results. One can see from the numbers given in Table 1 that a large increase is obtained as compared to the case \( N_{\text{max}} = 1 \), particularly when induced signals are needed (for example, from 8\% to 60\% in column C). However, the results concerning the case \( N_{\text{max}} = 2 \) (about 60–70\% when induced signals are considered) could still be improved as one can see from the study of the \( N_{\text{max}} = 3 \) case which allows to reconstruct up to 90\% of the total \( \gamma \)-events. This large gain can motivate a further PSA development up to the \( N_{\text{max}} = 3 \) case although this is presently an experimental challenge. This is actually the upper limit one should aim at, because the case \( N_{\text{max}} = 4 \) does not give a significant improvement of the tracking efficiency.

From the above discussion it is clear that, before any tracking procedure starts, one needs to disentangle the different pulses overlapping in a single segment. If such a disentangling procedure fails, then such an error will propagate to the position and energies given by the PSA which form the input list of the tracking algorithm. Therefore, such an error might affect the overall performance of the detector, independent of how sophisticated and accurate are both PSA and tracking.

\begin{table}[h]
\centering
\begin{tabular}{ccc}
\hline
\( N_{\text{max}} \) & A & B \\
\hline
1 & 48.5 & 14.6 & 8.4 \\
2 & 90.6 & 68.4 & 60.2 \\
3 & 98.8 & 92.1 & 87.5 \\
4 & 99.9 & 98.8 & 97.4 \\
\hline
\end{tabular}
\caption{Percentage of 1 MeV \( \gamma \)-events with \( N_s \leq N_{\text{max}} \) signals in all segments, for a 6 \( \times \) 6-segmented detector. The three columns correspond to the cases in which only the direct signal (column A), the direct and the two strongest induced signals (column B) or the direct and all the induced signals (column C) have to be taken into account by PSA.}
\end{table}
It is important to point out the fact that an event which has been wrongly localised by PSA does not necessarily make the tracking algorithm fail, especially if this happens in the last steps of the track. It is therefore useful to probe, by the use of simulations, the performance of the tracking algorithms in the conditions of incorrect input lists.

From the GEANT-simulated tracks relative to the MARS detector (the result will be essentially identical for similar segmented detectors), we have selected those events with \( N_s = 2, 3 \) or 4 and we have created three different input lists for the tracking algorithm. In the first (ideal list), all the interaction points have been correctly identified and localised. A position resolution of 1 mm and a resolving distance of 5 mm have been assumed. This is the typical input list used to test the tracking algorithms discussed, for example in Refs. [14–16]. In the second list (average list), if \( N_s > N_{\text{max}} \), then the energy of the different signals is summed up and a weighted position is extracted. In the third list (random list), if \( N_s > N_{\text{max}} \) then a random number of interaction points is chosen, from 1 to 4, being randomly positioned in the segment. The total energy is conserved and distributed randomly between the hits. In both average and random lists, only direct signals are summed up if they are superimposed in a given segment and it has been assumed that the superposition of induced signals over a direct signal does not disturb the determination of the individual interaction points.

It is interesting to notice that the case with \( N_{\text{max}} = 1 \) nearly corresponds to the situation discussed in the first section of this paper.

The very first consequence of the failure of the disentangling procedure is a loss of the total efficiency due to events that can no more be tracked. A tracking routine needs in fact at least two interaction points inside the detector. If the disentangling routine fails, then multiple events that are confined in a single segment are transformed into “one hit” events and cannot be tracked anymore. Table 2 summarises the percentage of the events, which cannot be tracked anymore for the “average” list and the “random” list. The numbers are about four times lower in the random case as the number of interactions is randomly chosen between 1 and 4, and consequently, only one-fourth of the confined events will be transformed into “one hit” events.

Both tracking algorithms studied here reject “one hit” events. Clearly, a final tracking procedure could simply accept these events as possible FEP events, consequently increasing the FEP tracking efficiency at the price of an increase of the Compton background.

The results of the one-\( \gamma \)-tracking and backtracking algorithms with the input lists previously discussed are displayed in Fig. 6 for reasonable choices of the relative figure of merit (\( w_{\text{upper}} = 70 \) and \( w_{\text{lower}} = 0.1 \)). The solid lines correspond to the ideal case, the dashed lines to the average case and the dotted lines to the random case. Clearly, for higher disentangling ability (higher \( N_{\text{max}} \)), the number of “wrong” events decreases and the tracking efficiency increases. In Fig. 7, only the subset of critical events having \( N_s > N_{\text{max}} \), is considered. About 60–80% of these \( \gamma \)-rays can still be correctly tracked by the one-\( \gamma \)-tracking algorithm, although position and energy are not well determined. The backtracking algorithm suffers dramatically from the random determination of the interaction points (dotted line). In fact, in this case, the physical information determining the photoelectric character of the last interaction has been lost, and the tracking algorithm fails.

Apart from this extreme case, approximately half of the events for which the interaction points have been incorrectly determined have been nonetheless correctly tracked, as shown in Fig. 7. These are probably the cases in which the photoelectric and the last Compton events are in the same segment.

<table>
<thead>
<tr>
<th>( N_{\text{max}} )</th>
<th>Average</th>
<th>Random</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2

Percentage of multiple hit \( \gamma \)-events which are confined in a single segment and become “one hit” events when the PSA fails, for different disentangling capacities, \( N_{\text{max}} \).
5. Conclusion

In this paper, we have used simulations to investigate the extent of the ability to reconstruct events with multiple interactions in a given segment of a segmented detector.

First, it is found that, at the simplest level, namely without any PSA or tracking, the segmentation itself improves the $P/T$ ratio, although at the price of a reduction of the efficiency. Moreover, for moving source measurements, one obtains a sizeable reduction of the Doppler broadening. It has also been shown that in this case, the use of a tracking algorithm does not help to further improve the performances of the detector.

Concerning the use of such a detector with PSA, we have focused on how a given tracking algorithm can recover events for which the single interaction points were not well identified. It is found that, with the actual degree of segmentation, most of the 1 MeV $\gamma$-events are characterised by the presence of a superposition of signals in some segments, which can be either direct or induced signals. It is therefore crucial that the PSA correctly identifies and separates the superimposed signals to avoid giving incorrect interaction points to the tracking procedure. However, the adopted tracking algorithms show the ability to recover in some cases the events with an incorrect position and energy determination. It is found that the best performance of PSA should be to identify three signals per segment, but this is a difficult challenge at present. The relevant conclusion, which is of importance as a guide for the current developments, is the identification of two signals per segment, which is a significant achievement because it ensures high performances of the tracking procedure.

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